Maplets for Calculus: Improving Student Skills and Understanding in Calculus

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Maplets for Calculus is a collection of 94 Maple applets for use by students and instructors in single-variable calculus. Students use them to develop intuition and to practice specific calculus skills. Instructors use them as lecture demonstrations and to create exam problems.

Maplets for Calculus (M4C) [1] is a collection of 94 Maple applets that are useful both for calculus students and for instructors. The maplets in M4C provide interactive graphical user interfaces for typical examples and exercises on a variety of topics in single-variable calculus. Most maplets allow for problems to be either algorithmically-generated or user-entered. The Maplets for Calculus balance the development of both understanding and technical skills. M4C is divided into the following eight categories:

- Pre-Calculus (22 maplets)
  - graphical properties of functions (translations, reflections, domain, range, asymptotes)
  - absolute value, quadratic, exponential, logarithmic and trigonometric functions
  - review of trig
- Limits (8 maplets)
  - conceptual understanding of limits and continuity, one-sided and two sided
  - ε-δ definition of the limit (graphical and formal proof)
  - intermediate value theorem and bisection method
- Derivatives (24 maplets)
  - basic principles: secant lines and tangent lines
  - practice with the terminology of derivatives and with making graphical connections between properties of a first or second derivative and the original function
  - drill and practice on differentiation techniques (chain rule, implicit differentiation, logarithmic differentiation, inverse functions)
  - applications (linear and quadratic approximations, related rates, motion, l'Hospital's rule, max/min problems)
  - antiderivatives and an introduction to the Fundamental Theorems of Calculus
- Integrals (28 maplets)
  - focused practice with specific integration techniques (substitution, integration by parts, trigonometric integrals, trigonometric substitution, partial fractions)
  - drills for definite and indefinite integration
  - geometrical applications (area of a region, volume by slicing, volume of revolution, arc length, surface area)

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[1] The Maplets for Calculus are for sale to individuals at Maplesoft's webstore at http://www.maplesoft.com/products/thirdparty/maplets%5Fcalc/ Contact the authors about site licenses.

- o physical applications (average value, mass, center of mass, work, fluid force)
- o numeric integration (left and right Riemann sums, trapezoid and Simpson's rules)
- Differential Equations (4 maplets)
  - o drills for both separable and linear differential equations
  - o direction fields
  - o mixing problems
- Sequences and Series (4 maplets)
  - o general drills for limits of sequences and convergence of series
  - o more focused practice with geometric and telescoping series
- Curvilinear Coordinates (2 maplets)
  - o identification of polar curves
- Games (2 maplets)
  - o plotting and cryptograms

Students find the algorithmically-generated problems very helpful for practicing new techniques and for building confidence as they prepare for a quiz or exam. Immediate feedback and infinite patience are two features students find useful. Students find the hints and step-by-step checking of their responses as effective as a private tutor. For example, Figure 1 shows the Integration by Substitution maplet in which the user has omitted the differential in the substitution.

Instructors have multiple opportunities to use the Maplets for Calculus to enhance their courses. The ability to launch a Maplet with a specific example makes it convenient to use a Maplet for demonstrations and examples in a lecture. Plus, the extensive use of graphics – 2D, 3D, and animation – add to the impact that the Maplets can have in the classroom. Examples include the maplets: Epsilon Delta Definition of a Limit (Figure 2), Derivatives of Inverse Functions (Figure 3), Volume by Slicing (Figure 4) and Area as an Antiderivative (Figure 5). Others will find it more appropriate to develop a project or a lab that utilizes a Maplet in a guided-discovery exercise. For example, the Epsilon Delta Definition of a Limit maplet provides a graphical and interactive environment for exploring the ε-δ definition of a limit (Figure 2). Instructors can also have their students drill on difficult topics such as Indefinite or Definite Integration Techniques and Series Convergence Tests (Figure 6 — Figure 9).

A Table of Contents and sample videos of the Maplets for Calculus may be seen at http://m4c.math.tamu.edu/. An expanded and updated version of Maplets for Calculus was completed in January 2008. The remainder of this paper shows snapshots taken from several different maplets in the M4C collection.
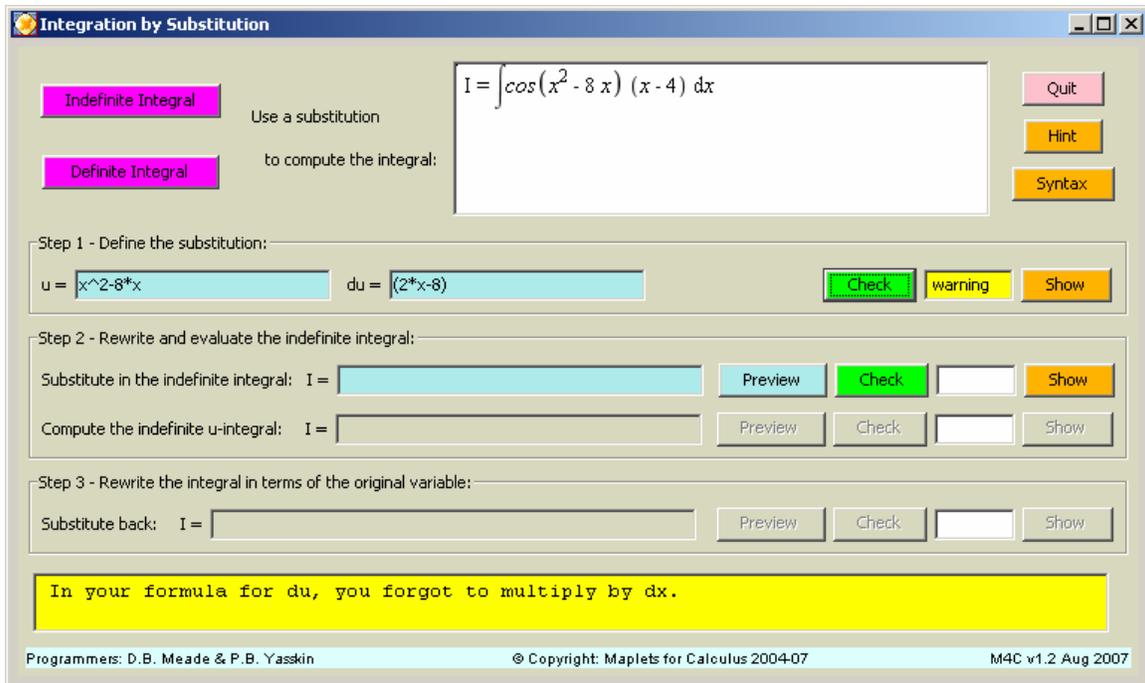
Figure 1: Constructive feedback in the Integration by Substitution maplet. Note that the user has omitted the differential (*dx*) in their response for *du*.
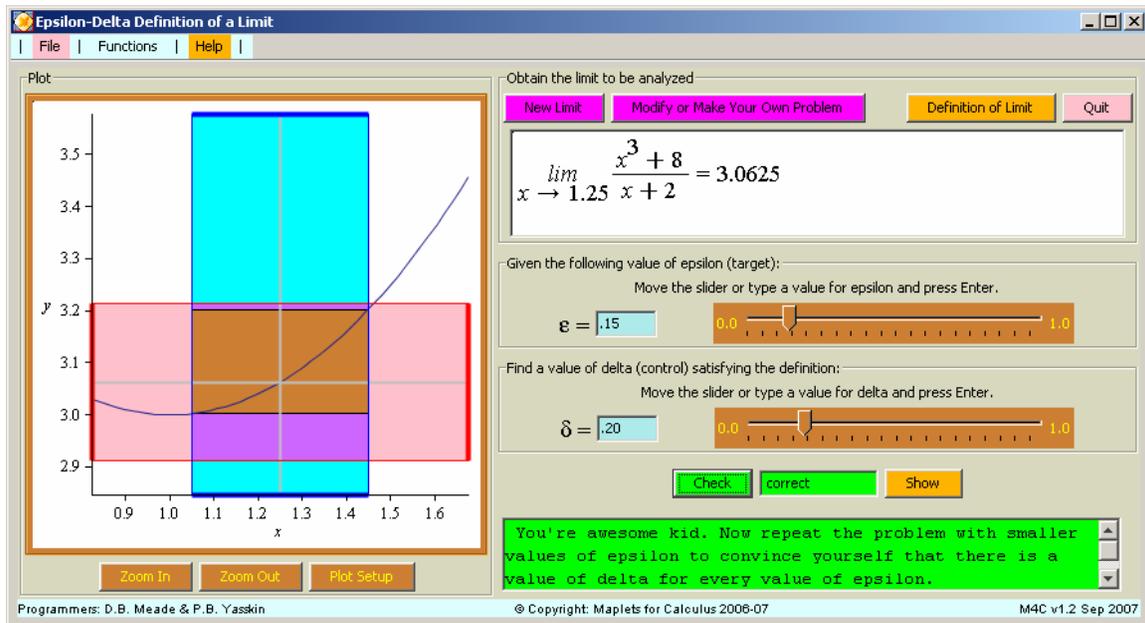

Figure 2: Visualizing the ε-δ definition of the limit with the Epsilon Delta Definition of a Limit maplet. Users can control the values of ε and δ using the sliders, or by entering a value in the corresponding input field.
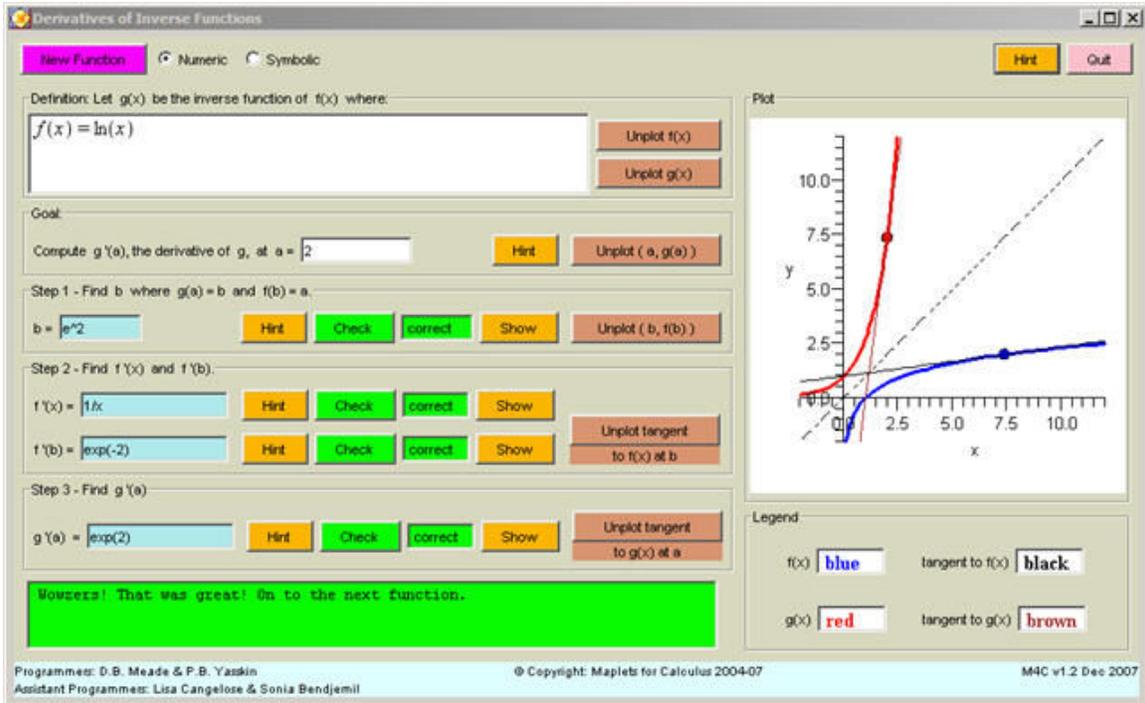
Figure 3: The Derivatives of Inverse Functions maplet, showing a function and its inverse along with tangent lines at mirror image points.
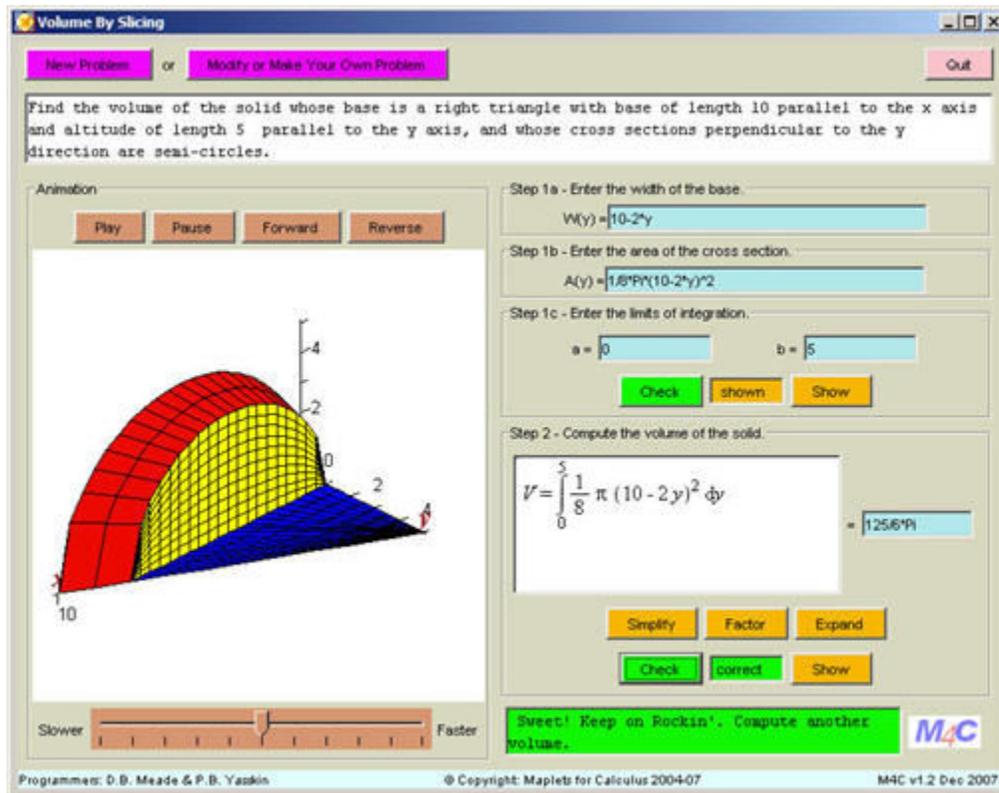
Figure 4: The Volume By Slicing maplet, showing one frame of the 3D animation and the emphasis on the setup of the definite integral.
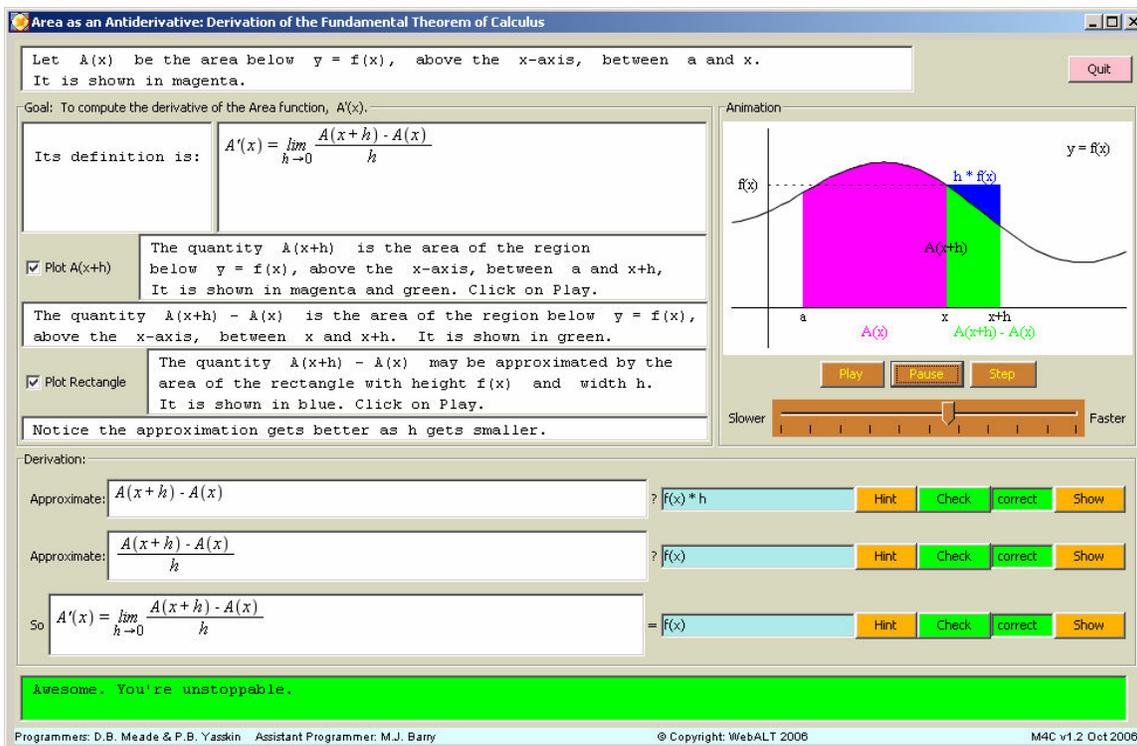
Figure 5: In the Area as an Antiderivative: Derivation of the Fundamental Theorem of Calculus maplet an animation helps students understand why the FTC is true.
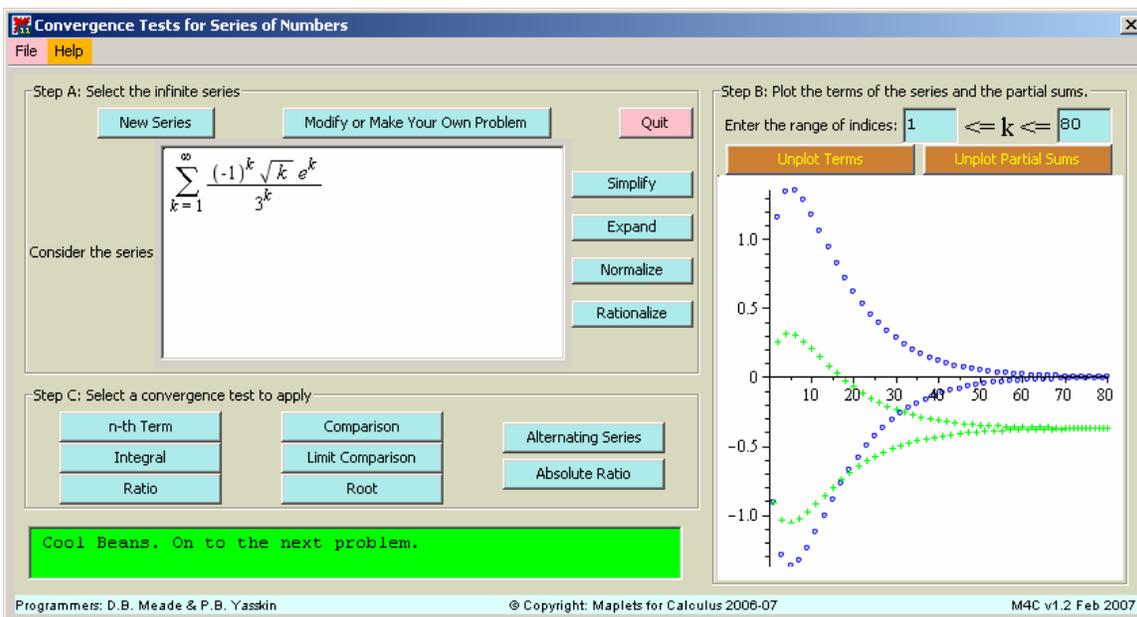

Figure 6: The Convergence Tests for Series of Numbers maplet. When users select a convergence test to apply, the test appears in a separate popup window. (See the next 3 figures.) Note the graphical display of the terms (blue) and the partial sums (green).
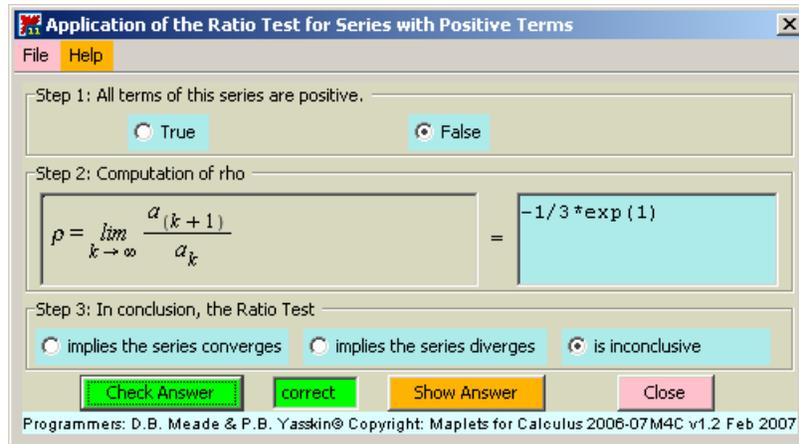
Figure 7: The popup window for the Ratio Test. This test is inconclusive because the terms of this series are not all positive.
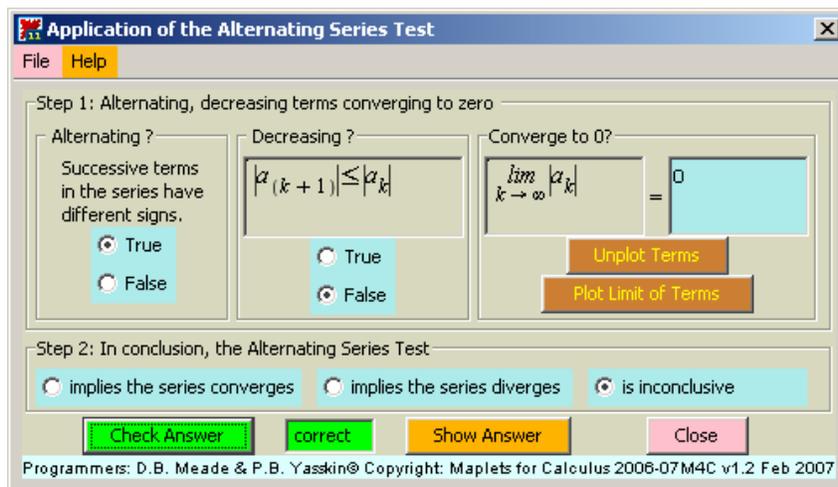

Figure 8: The popup window for the Alternating Series Test. This test cannot be applied because the sequence of terms increases (in absolute value) for the first few terms.
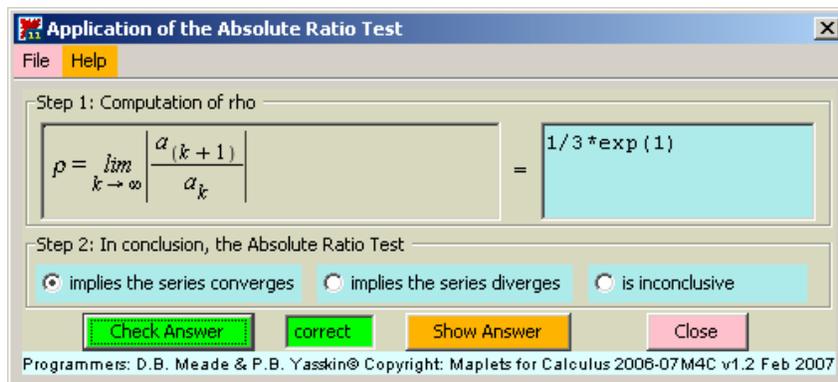

Figure 9: The popup window for the Absolute Ratio Test. This test can be used to show the convergence of this series.